DIRECT AND ITERATIVE ALGORITHMS IN SPECTRAL METHODS FOR BOUNDARY VALUE PROBLEMS INVOLVING SYSTEMS OF ODE

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Spectral methods are widely used for solving differential problems involving PDE and systems of ODEs. In the most cases (may be, with exception of the Fourier method) implementation of spectral method is related with processing systems of algebraic equations having a specific block structure of the system matrix. In the case of multidimensional DPE and large systems of ODE, corresponding discrete problems on the base of spectral method results in algebraic systems with sufficiently sparse matrix (all blocks with exception of the main diagonal have diagonal form). Our goal is to compare efficiency of direct and iterative methods applying to solving systems of linear algebraic equations arising in spectral method for boundary value problems with large systems of ODE. As an example, the boundary value problem for modeling optical fiber amplifier is considered [1]:

$$M\frac{dE}{dz} = -[\gamma - G(E)]E, \ -1 < z < 1, \tag{1}$$

where $E = (E_1, E_2, ..., E_n)^T$, E_k is complex envelopes of light wave magnitudes, M is a diagonal matrix with entries $\{M_{kk}\} = \pm 1$, sign of which is defined by propagation direction of the corresponding wave component, $G(E)$ is a matrix $n \times n$ defining nonlinear interaction of the components $\{G_{km}\} = g_{km}E_m^*E_k$, with coefficients $g_{km} = -g_{mk}$ dependent on difference in the component frequencies, γ is the adsorption coefficient. The boundary conditions for each of the components are specified at the left or right side of the interval in dependence on the propagation direction of corresponding wave.

Discretization of problem (1) using Chebyshev mesh $z_j = \cos\frac{j\pi}{N-1}$ results in a system of nonlinear equations $Nn \times Nn$:

$$(\overline{D} + \gamma - \overline{G}(U))U = F, \tag{2}$$

where $\overline{G}$ is a block matrix with diagonal blocks $\{\overline{G}_{km}\} = g_{km}\operatorname{diag}\left(U_m^*(z_j)U_k(z_j)\right)$, $j = 1, 2, \ldots, N$, $\overline{D} = M \otimes D$ is a block diagonal matrix composed of the Chebyshev spectral differentiation matrix D with modified first or last row according to the boundary conditions defined by the right hand side vector F.

For linearization of the system (2) we used two iterative methods. One of the methods is the Newton method:

$$U^{(k+1)} = U^{(k)} - pJ^{-1}\left((\overline{D} + \gamma - \overline{G}(U^{(k)}))U^{(k)} - F\right), \tag{3}$$

where J is the Jacobian, p is the iterative parameter. The second method has the form similar to the iterative technique used in ref. [2]:

$$(\overline{D}+\gamma-\overline{G}(U^{(k)}))U^{(k+1)}=F. \qquad (4)$$

In the both cases iterative methods (3) and (4) result in a system of linear algebraic equations with matrices having similar block structure. For solving the obtained equations we used the direct Gauss elimination method and iterative methods provided by the Matlab standard functions. For the considered problem the best performance is demonstrated by the Generalized Minimum Residual iterative method with Block-Jacoby preconditioner in the form of the matrix $\overline{D}$. Simulation results with estimation of the calculation time for solving the problem using direct and iterative methods are presented in Fig. 1 below.

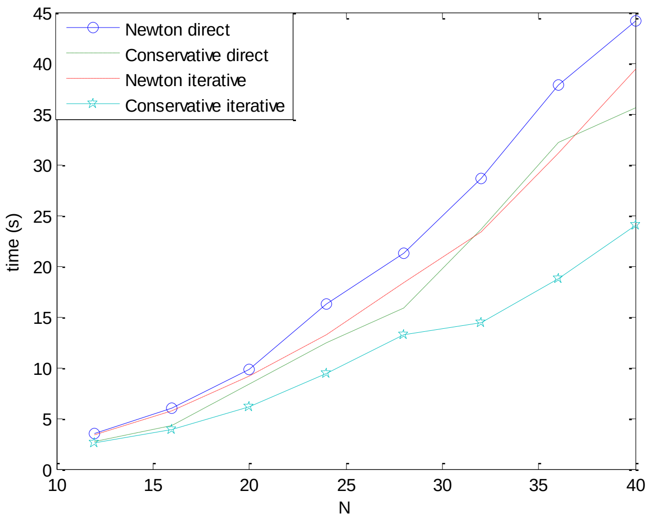

Fig. 1. Dependences of the calculation time on grid size to solve the problem by means of direct and iterative methods.

It can be seen in Fig., the calculation time grows more faster with increasing grid size in the case when the direct method for solution of the problem is used. Note, the iterative implementation of the linear problem is more efficient in the case when iterative method (4) is used for linearization of the problem. The presented results are obtained for the problem with six ODE, $n=6$. For the case of larger systems, $n>10$ the advantages of the iterative implementations become more impressive. As a conclusion, we can emphasise that for tipical problems in design of optical fiber amplifiers for WDM optical communication systems where the number of simulated components is about a few hundred the iterative methods are prove to be more efficient in comparison with the methods of direct type.

References

1. Headley, C. Raman Amplification in Fiber Optical Communication Systems / C. Headley, G.P. Agrawal – San Diego: Academic Press, 2005. – 376p.